1.4 - MAY'S THEOREM Mathematician: ________________________

How can we prove an important theorem about social choice?

First, let's look at a voting profile and determine the winner using different rules.

1. Consider the voting profile

A	A	B	A	B	A	A	B	B	B	B	B	B	A	B	B	B

Who wins under:
(a) the simple majority method?

(b) the super-majority method with p = 2/3 (p is the threshold for victory)?

(c) the super-majority method with p = 2/3 but with A as the status quo?

(d) the parity method?

(e) minority rule?

2. Suppose you favor one of two candidates but only 10% of the electorate agrees with your position. Is there a voting method that leads to victory for your position that is

(a) anonymous?

(b) neutral?

(c) anonymous and neutral?

(d) monotone?

(e) anonymous, neutral, and monotone?

3. May's Theorem states the following: in a two-candidate election, the only system that is anonymous, neutral, monotone, and nearly decisive is simple majority.

To prove this, we need to do a **proof by contradiction**. This is a common mathematical strategy where you assume something is true and then show that that's impossible, and so thus it must be false (and the opposite is true).

First, suppose we have a voting system that is anonymous, neutral, monotone, and nearly decisive. Because of ___________________, we only need to consider total votes and not individual ballots.

Let a be the number of votes for candidate A, and b be the number of votes for candidate B, so that $t =$ ________ is the total number of votes.

Suppose that t is even. If $a = b = \frac{1}{2}t$, then there is a tie, because our system is ___________________. (We could show this more explicitly, but I think this is fine for our purposes.)

Now suppose that A has a majority, which means that $a \geq$ ____________. We need to show that A is a unique winner. By ___________________ we know there can't be a tie in this situation, so we just need to show that B can't be a winner.

Suppose that B were a winner. In that case,

$b =$ __________ ≤ ____________ = ______________ < ______________

If B is a winner, then, by _____________, that would mean that if B gained votes, they would remain a winner. However, because $b <$ ________, that means B could gain votes so that $b =$ _________. But we previously stated that, in that situation, there's a tie. Contradiction! That means B can't be a winner.

By ____________, we can switch A and B in all of our calculations to show the reverse. Thus this system with all our stated properties is equivalent to simple majority.